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Wavelets and Signal Processing

PROGRESS REPORT

Progress report for the period 1 October 1995 to 15 September 1996

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1 Objectives

(1) Malvar wavelets have been recognized in signal and image processing to be useful for eliminating the blocking effects in signal coding. Many researchers have contributed to this area for scalar-valued Malvar wavelets. The Shannon information theory suggests that vector quantization is, in general, better than scalar quantization for data compression. As such, the objectives of this effort were to investigate vector-valued Malvar wavelets. Specifically, we are studying vector-valued function spaces and their properties, vector-valued local bases, and implementations of vector-valued Malvar wavelets with vector filter banks.

(2) Orthonormal bases of wavelets $\psi_{j,k}(x) = 2^{-j/2}\psi(2^{-j}x - k)(j, k \in \mathbb{Z})$ for $L^2(\mathbb{R})$ have many useful properties. The construction of such wavelet bases is well understood; every such basis corresponds to a multiresolution analysis characterized by a *scaling function* $\phi(x)$. What makes a randomly chosen ϕ a scaling function? There exist many different possible choices for ϕ. We try to point out that scaling functions can all be viewed as the fixed points of a nonlinear operator.

(3) We introduce the *new* concept of the Fractional Wave Packet Transform(FRWPT), based on the idea of the Fractional Fourier Transform(FRFT) and Wave Packet Transform(WPT). We show a version of the resolution of the identity and some properties of FRWPT connected with those of FRFT and WPT.

2 Status of Effort

We introduced vector-valued multiresolution analysis and vector-valued wavelets for vector-valued signal spaces. We constructed vector-valued wavelets using paraunitary vector filter bank theory. We classified and constructed vector-valued wavelets with the sampling property. As an application of vector-valued wavelets, multiwavelets can be constructed from vector-valued wavelets. We showed that linear combinations of scalar-valued wavelets yield multiwavelets. In addition, by applying proper prefiltering, data compression gains can be achieved over scalar-valued wavelets. Because of the significance of this data compression result, WL/ELED is now planning to fabricate a multiwavelet chip during FY97.

We also studied a certain nonlinear operator T from $L^2(\mathbb{R})$ to itself under which every

scaling function(vector) is a fixed point. The iterations $T^n f$ of T on any L^2-function f with the Riesz basis property are investigated; they turn out to be the subdivision scheme iterates of f with weights depending on f only. We give conditions for convergence of $T^n f$ to a limit in different topologies and studies the regularity of the limit functions. The results are illustrated with examples.

We have introduced the concept of the Fractional Wave Packet Transform(FRWPT), combining the idea of the Fractional Fourier Transform(FRFT) and Wave Packet Transform(WPT). We have obtained a version of the resolution of the identity and some properties of FRWPT connected with those of FRFT and WPT. Further study can be on the relationships of the FRWPT with other time-frequency representations such as Wigner distribution, the ambiguity function and the spectrogram.

3 Accomplishments/New Findings

The accomplishments given in this report are in the following areas: (1) Design of prefilters for discrete multiwavelet transform, (2) Vector-valued wavelets and vector filter banks, (3) A nonlinear operator Related to wavelets and refinable function vector and (4) Fractional wave packet transform. These results provide fundamental tools for many Air Force and industrial applications, including the compression of images/video images.

A. Design of Prefilters for Discrete Multiwavelet Transform

The pyramid algorithm for computing wavelet transform coefficients is well known. This algorithm can be implemented using tree-structured multiwavelet filter banks. A general algorithm was presented to compute the multiwavelet transform coefficients by adding conventional multirate filter bank as a prefilter before the vector filter banks that generate multiwavelets. This algorithm can be thought of as a discrete vector-valued transform for certain discrete-time vector-valued signals. Also, numerical experiments have been performed that illustrate the performance of the algorithm, which indicates that energy compaction for discrete multiwavelets will be better than the energy compaction for conventional discrete wavelet transforms. As a test signal, the 100th line of the cameraman image was used. Let the energy compaction ratio be defined as the energy in the bandpass and highpass parts over the total energy of the signal. The Daubechies wavelet transform D_4 had an energy

compaction ratio of 11%, while the multiwavelet with prefiltering had an energy compaction ratio of 5%.

B. Vector-Valued Wavelets and Vector Filter Banks

We introduced vector-valued multiresolution analysis and vector-valued wavelets for vector-valued signal spaces. Vector-valued wavelets using paraunitary vector filter bank theory were constructed. We classified and constructed vector-valued wavelets with a sampling property. Discrete vector wavelet transforms for discrete-time vector-valued signals were also generated.

C. A Nonlinear Operator Related to Wavelets and Refinable Function Vectors

It is now well known that every wavelet basis $\psi_{j,k}(x) = 2^{-j/2}\psi(2^{-j}x - k)$ $(j, k \in \mathbb{Z})$ for $L^2(\mathbb{R})$ corresponds to a multiresolution analysi s characterized by a *scaling function* $\phi(x) \in L^2(\mathbb{R})$. Scaling functions are special cases of refinable functions, i.e. they satisfy a refinement equation,

$$\phi(x) = \sum_{k\in\mathbb{Z}} c_k \phi(2x - k),$$

where the set of coefficients $\{c_k\}$ is called a *mask* or filter. Recently, several authors have considered generalizations of the above equation with more than one function ϕ; in this case one has the following N equations:

$$\Phi(x) = \sum_{k\in\mathbb{Z}} C_k \Phi(2x - k),$$

where Φ is a vector, again plays important role in the *multiresolution analysis of mult iplicity* N. This generalization also leads to interesting new constructions, such as orthogonal wavelet bases using fractal interpolation functions. We study a certain nonlinear operator $\mathcal{T}$ under which every refinable function vector is a fixed point. This operator $\mathcal{T}$ projects any nonzero function vector onto the closed subspace spanned by its own scaled translates. More precisely, for any $\mathbf{f} = (f^1, ..., f^N)^*$:

$$\mathcal{T} f^r \;:=\; \text{Orthogonal Projection of } f^r \text{ onto } \overline{\mathrm{Span}\{f^r_{-1,k}; k \in \mathbb{Z}, \mathrm{r} = 0, 1, ..., \mathrm{N}-1\}},$$

where as usual, $f^{r}{}_{j,k} = 2^{-j/2} f^{r}(2^{-j}x - k)$. It is clear that $\mathcal{T}\Phi = \Phi$, and

$$\|\mathcal{T}\mathbf{f}\| \leq \|\mathbf{f}\|,$$

The operator $\mathcal{T}$ is nonlinear: though it is obvious that $\mathcal{T}(\lambda\mathbf{f}) = \lambda\,\mathcal{T}\mathbf{f}$, for all $\lambda \in \mathbb{C}$, we have explicit counterexamples for which $\mathcal{T}(\mathbf{f} + \mathbf{g}) \neq \mathcal{T}\mathbf{f} + \mathcal{T}\mathbf{g}$.

We want to study the iterations $\mathcal{T}^n\mathbf{f}$ of $\mathcal{T}$ on an arbitrary function vector $\mathbf{f}$. If these $\mathcal{T}^n\mathbf{f}$ h ave a nontrivial limit, then this limit is a fixed point of $\mathcal{T}$ and is therefore a refinable function vector. It leads to a multiresolution analysis of multiplicity N "naturally" associat ed with $\mathbf{f}$.

D. Fractional Wave Packet Transform

We have introduced the concept of the Fractional Wave Packet Transform(FRWPT), combining the idea of the Fractional Fourier Transform(FRFT) and Wave Packet Transform(WPT). We have obtained a version of the resolution of the identity and some properties of FRWPT connected with those of FRFR and WPT. Further study can be on the relationships of the FRWPT with other time-frequency representations such as Wigner distribution, the ambiguity function and the spectrogram.

4 Personnel Supported

Dr. Bruce Suter - principal investigator

Dr. Mark Oxley - co-principal investigator

Dr. Ying Huang - post-doctoral researcher

5 Publications

A. Publications(Appeared or Accepted)

[P1] X.-G. Xia and B. W. Suter, *On the Householder Transform in C^m*, DIGITAL SIGNAL PROCESSING, 5, 116–117 (1995).

[P2] X.-G. Xia, B. W. Suter, and M. E. Oxley, *On Necessary and Sufficient Conditions for Perfect Reconstruction Multidimensional Delay Chain Systems*, IEEE TRANSACTIONS ON SIGNAL PROCESSING, 43, 1515–1519 (1995).

[P3] X.-G. Xia and B. W. Suter, *A Family of Two Dimensional Nonseparable Malvar Wavelets*, APPLIED AND COMPUTATIONAL HARMONIC ANALYSIS, 2, 243–256 (1995).

[P4] X.-G. Xia and B. W. Suter, *On Vector Karhunen-Loeve Transforms and Optimal Vector Transforms*, IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS FOR VIDEO TECHNOLOGY, 5, 372–374 (1995).

[P5] J. R. O'Hair and B. W. Suter, *A New Methodology for Developing Fast Algorithms Using Multirate*, IEEE TRANSACTIONS ON SIGNAL PROCESSING, 43, 3042–3046 (1995).

[P6] X.-G. Xia, J. S. Geronimo, D. P. Hardin, and B. W. Suter, *Design of Prefilters for Discrete Multiwavelet Transforms*, IEEE TRANSACTIONS ON SIGNAL PROCESSING, 44, 1099–1110 (1996).

[P7] X.-G. Xia and B. W. Suter, *Vector-Valued Wavelets and Vector Filter Banks*, IEEE TRANSACTIONS ON SIGNAL PROCESSING, 44, 508–518 (1996).

[P8] X.-G. Xia and B. W. Suter, *FIR Paraunitary Filter Banks Given Several Analysis Filters: Factorizations and Constructions*, IEEE TRANSACTIONS ON SIGNAL PROCESSING, 44, 720–723 (1996).

[P9] X.-G. Xia, B. W. Suter, and M. E. Oxley, *Malvar Wavelets with Asymmetrically Overlapped Windows*, IEEE TRANSACTIONS ON SIGNAL PROCESSING, 44, 723–728 (1996).

[P10] J. R. O'Hair and B. W. Suter, *The Zak Transform and Decimated Time-Frequency Distributions*, IEEE TRANSACTIONS ON SIGNAL PROCESSING, 44, 1099–1110 (1996).

[P11] Y. Huang, *A nonlinear operator related to wavelets and scaling functions*, *SIAM J. Math.*, to appear in November 1996.

[P12] I. Daubechies and Y. Huang,*How does truncation of the mask affect a refinable function?* CONSTR. APPROX., 11, 365–380 (1995).

[P13] A. V. Lair and M. E. Oxley, *Anisotropic Nonlinear Diffusion with Absorption: Existence and Extinction*, INTERNATIONAL JOURNAL OF MATHEMATICS AND MATHEMATICAL SCIENCES, 19, 427–434 (1996).

[P14] T. J. Burns, S. K. Rogers, M. E. Oxley and D. W. Ruck, *A Non-Homogeneous Wavelet Multiresolution Analysis for Spatio-temporal Signals*, IEEE TRANSACTIONS ON AEROSPACE AND ELECTRONIC SYSTEMS, 32, 628–649, (1996).

[P15] C. A. Bleckmann, M. E. Oxley, E. J. Wilson, K. W. Hayes, and N. L. Hercyk, *Land Treatment of Produced Oily Sand: Field Results*, WASTE MANAGEMENT RESEARCH, accepted March 1996.

B. Publications Submitted

[S1] Y. Huang and B. W. Suter, *Fractional Wave Packet Transforms*, MULTIDIMENSIONAL SYSTEMS AND SIGNAL PROCESSING.

[S2] Y. Huang and B. W. Suter, *A Study of Refinable Function Vectors via a Nonlinear Operator*, CONSTRUCTIVE APPROXIMATION.

6 Interactions/Transitions

A. Participation/Presentations at Conferences, Meetings, Seminars

[C1] X.-G. Xia and B. W. Suter, *Malvar Wavelets on Hexagons*, at the INTERNATIONAL CONFERENCE ON APPROXIMATION THEORY, College Station, Texas (1995).

[C2] X.-G. Xia and B. W. Suter, *On Vector-Valued Orthogonal Wavelets*, in H. Szu (editor), PROCEEDINGS OF SPIE CONFERENCE ON WAVELET APPLICATIONS II, 2491, 903–914, Orlando, Florida (1995).

[C3] X.-G. Xia, J. S. Geronimo, D. P. Hardin, and B. W. Suter, *On Computations of Multiwavelet Transforms*, in A. Laine and M. Unser (editors), PROCEEDINGS OF SPIE CONFERENCE ON WAVELET APPLICATIONS IN SIGNAL AND IMAGE PROCESSING III, 2569, 27–38, San Diego, California (1995).

[C4] X.-G. Xia, B. W. Suter, and Y. Huang, *Vector Malvar Wavelets* in J. Picone (editor), PROCEEDINGS OF SPIE CONFERENCE ON DIGITAL SIGNAL PROCESSING TECHNOLOGY, 2750, 129–138, Orlando, Florida (1996).

[C5] D. A. McCandless, S. K. Rogers, J. W. Hoffmeister, D. W. Ruck, R. A. Raines, and B. W. Suter, *Wavelet Detection of Clustered Microcalcifications* in H. Szu (editor), PROCEEDINGS OF SPIE CONFERENCE ON WAVELET APPLICATIONS III, 2762, 388–399, Orlando, Florida (1996).

[C6] X.-G. Xia, J. S. Geronimo, D. P. Hardin, and B. W. Suter, *Why and How Prefiltering of Discrete Multiwavelet Transforms*, PROCEEDINGS OF THE IEEE INTERNATIONAL CONFERENCE ON ACOUSTICS, SPEECH, AND SIGNAL PROCESSING, Atlanta, Georgia (1996).

[C7] Y. Huang and B. W. Suter, *A Study of Refinable Function Vectors via a Nonlinear Operator*, PROCEEDINGS OF THE IEEE INTERNATIONAL SYMPOSIUM ON TIME-FREQUENCY AND TIME-SCALE ANALYSIS, 297–300, Paris, France (1996).

[C8] Y. Huang and B. W. Suter, *Nonlinear Operator Related to Refinable Function Vectors* at the SPIE CONFERENCE ON WAVELET APPLICATIONS IN SIGNAL AND IMAGE PROCESSING, Denver, Colorado in August 1996.

[C9] Y. Huang and B. W. Suter, *Fractional Wavelet Packet Transforms* at the IEEE SIGNAL PROCESSING WORKSHOP, Leon, Norway in September 1996.

[C10] M. A. Carter and M. E. Oxley, *Combinatorial Geometry and Vapnik-Chervonenkis Dimension*, PROCEEDINGS OF SPIE, paper no. 2824-32, accepted January 1996.

[C11] M. A. Carter and M. E. Oxley, *Combinatorial Geometry and Vapnik-Chervonenkis Dimension*, PROCEEDINGS OF THE WORLD CONGRESS ON NEURAL NETWORKS 1996, accepted January 1996.

B. Transitions Many Air Force and commercial applications will benefit from these results. From the commercial world of High Definition Television to the Air Force world of reconnaissance, the ability efficiently transmit video images is a critical technology.

As a result of continuing CAD tool problems, WL/ELE [contact: Dr. Robert Ewing (513) 255-7438] delayed the development of the Malvar wavelet chip. It is now scheduled to be completed in early 1997.

7 New Discoveries, Inventions, or Patents

None

8 Honors/Awards

Dr. Suter continues to serve as Associate Editor of the IEEE TRANSACTIONS ON SIGNAL PROCESSING.

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Key scientific results in the following four areas: (1) design of prefilters for discrete multiwavelet transform, (2) vector-valued wavelets and vector filter banks, (3) a nonlinear operator related to wavelets and refinable function vector and (4) fractional wave packet transform. These results provide fundamental tools and theories for many Air Force and industrial applications, including compression of images/video images.

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